Differentially Private Markov Chain Monte Carlo

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FCAI Finnish Center for Artificial Intelligence

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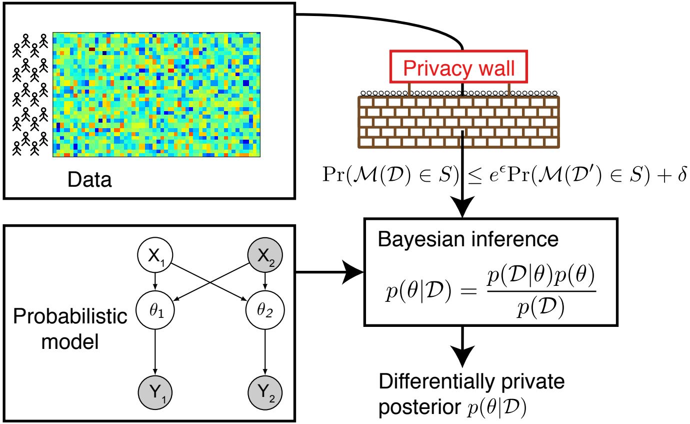
Motivation


Data
Privacy wall
$\Pr(\mathcal{M}(\mathcal{D}) \in S) \leq e^{\epsilon}\Pr(\mathcal{M}(\mathcal{D}') \in S) + \delta$
Probabilistic model
X_1
X_2
θ_1
θ_2
Y_1
Y_2
Bayesian inference
$p(\theta|\mathcal{D}) = \frac{p(\mathcal{D}|\theta)p(\theta)}{p(\mathcal{D})}$
Differentially private posterior $p(\theta|\mathcal{D})$

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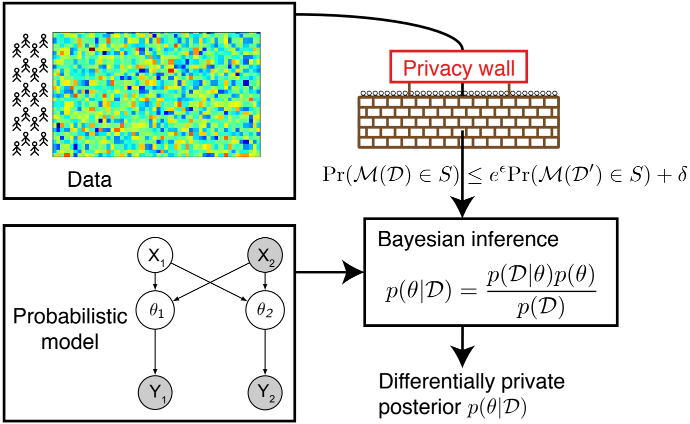

We propose a method for sampling from posterior distribution under DP guarantees.

DP mechanisms for Bayesian inference

Three general purpose approaches for DP Bayesian inference:

1. Drawing single samples from the posterior with the exponential mechanism (Dimitrakakis *et al.*, ALT 2014; Wang *et al.*, ICML 2015; Geumlek *et al.*, NIPS 2017)

   Privacy is conditional to sampling from the true posterior.

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2. Perturbation of gradients in SG-MCMC (Wang *et al.*, ICML 2015, Li *et al.*, AISTATS 2019) or variational inference (Jälkö *et al.*, UAI 2017) with Gaussian mechanism, similar to DP stochastic gradient descent

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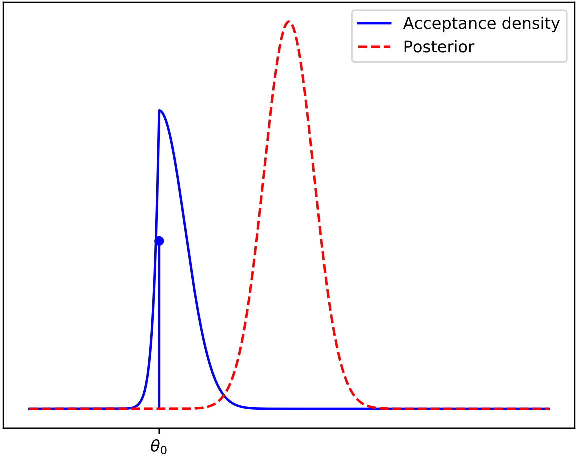
3. Computing the privacy cost of Metropolis–Hastings acceptances for the entire MCMC chain (Heikkilä *et al.*, NeurIPS 2019; Yıldırım & Ermiș, Stat Comput 2019)

Intuition



We employ the stochasticity of this decision to assure privacy

Outline of the method

Acceptance test (Barker et al. 1965)

Accept θ' from proposal q if $\Delta(\theta'; \mathcal{D}) + V_{logistic} > 0$

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Decompose the logistic noise : $V_{logistic} = V_{normal} + V_{correction}$

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Analyse the privacy implications (**This work**)

We use Rényi DP to compute the privacy guarantees of the acceptance condition

Subsampling allows us to benefit from privacy amplification (Wang *et al.*, AISTATS 2019)

Conclusions

- We have formulated a DP MCMC method for which privacy guarantees do not rely on the convergence of the chain.

Come see us at our poster **#158** in **East Exhibition Hall (B + C)**

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