

Learning Loop Invariants for Program Verification

Xujie Si*, Hanjun Dai*, Mukund Raghothaman, Mayur Naik, Le Song



University of Pennsylvania
Georgia Institute of Technology



NeurIPS 2018

Code: <https://github.com/PL-ML/code2inv>

* equal contribution

Program verification

- Prove whether your code is bug-free

Program verification

- Prove whether your code is bug-free

-- Some of rules can be automated:

sequence rule, conditional rule,

Program verification

- Prove whether your code is bug-free

-- Some of rules can be automated:

sequence rule, conditional rule,

-- Except 'while rule'

Loop Invariant <> Halting Problem

What is loop invariant?

What is loop invariant?

Program

```
 $x := -50;$   
while ( $x < 0$ ) {  
     $x := x + y;$   
     $y := y + 1$  }  
assert( $y > 0$ )
```

What is loop invariant?

Program

```
 $x := -50;$   
while  $(x < 0)$  {  
     $x := x + y;$   
     $y := y + 1$  }  
assert  $(y > 0)$ 
```

Loop Invariant

$(x < 0 \vee y > 0)$

What is loop invariant?

Program

```
 $x := -50;$   
while  $(x < 0)$  {  
   $x := x + y;$   
   $y := y + 1$  }  
assert  $(y > 0)$ 
```

Loop Invariant

$$(x < 0 \vee y > 0)$$

Requirement:

$$\forall x, y: \begin{cases} \text{true} \Rightarrow I[-50/x] & (pre) \\ I \wedge x < 0 \Rightarrow I[(y+1)/y, (x+y)/x] & (inv) \\ I \wedge x \geq 0 \Rightarrow y > 0 & (post) \end{cases}$$

Loop Invariant Checker

```
 $x := -50;$   
while ( $x < 0$ ) {  
     $x := x + y;$   
     $y := y + 1$  }  
assert( $y > 0$ )
```

$(x < 0 \vee y > 0)$

Loop Invariant Checker

```
 $x := -50;$   
while ( $x < 0$ ) {  
   $x := x + y;$   
   $y := y + 1$  }  
assert( $y > 0$ )
```

$(x < 0 \vee y > 0)$



Z3

Loop Invariant Checker

```
 $x := -50;$   
while ( $x < 0$ ) {  
   $x := x + y;$   
   $y := y + 1$  }  
assert( $y > 0$ )
```

$(x < 0 \vee y > 0)$

Z3



Difficulties of learning loop Invariant

1. Highly sparse and non-smooth reward

```
int main() {  
    // variable declarations  
    int x;  
    int y;  
    // pre-conditions  
    (x = 1);  
    (y = 0);  
    // loop body  
    while ((y < 1000)) {  
        {  
            (x = (x + y));  
            (y = (y + 1));  
        }  
    }  
    // post-condition  
    assert( (x >= y) );  
}
```

code

Difficulties of learning loop Invariant

1. Highly sparse and non-smooth reward

$$(x < 0 \vee y > 0)$$

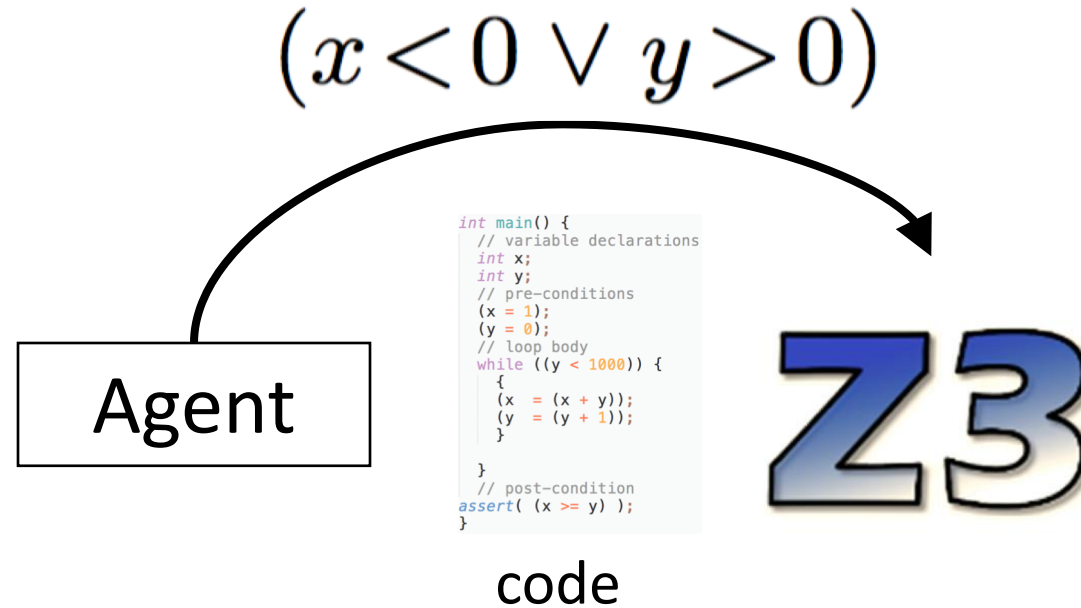
Agent

```
int main() {  
    // variable declarations  
    int x;  
    int y;  
    // pre-conditions  
    (x = 1);  
    (y = 0);  
    // loop body  
    while ((y < 1000)) {  
        {  
            (x = (x + y));  
            (y = (y + 1));  
        }  
    }  
    // post-condition  
    assert( (x >= y) );  
}
```

code

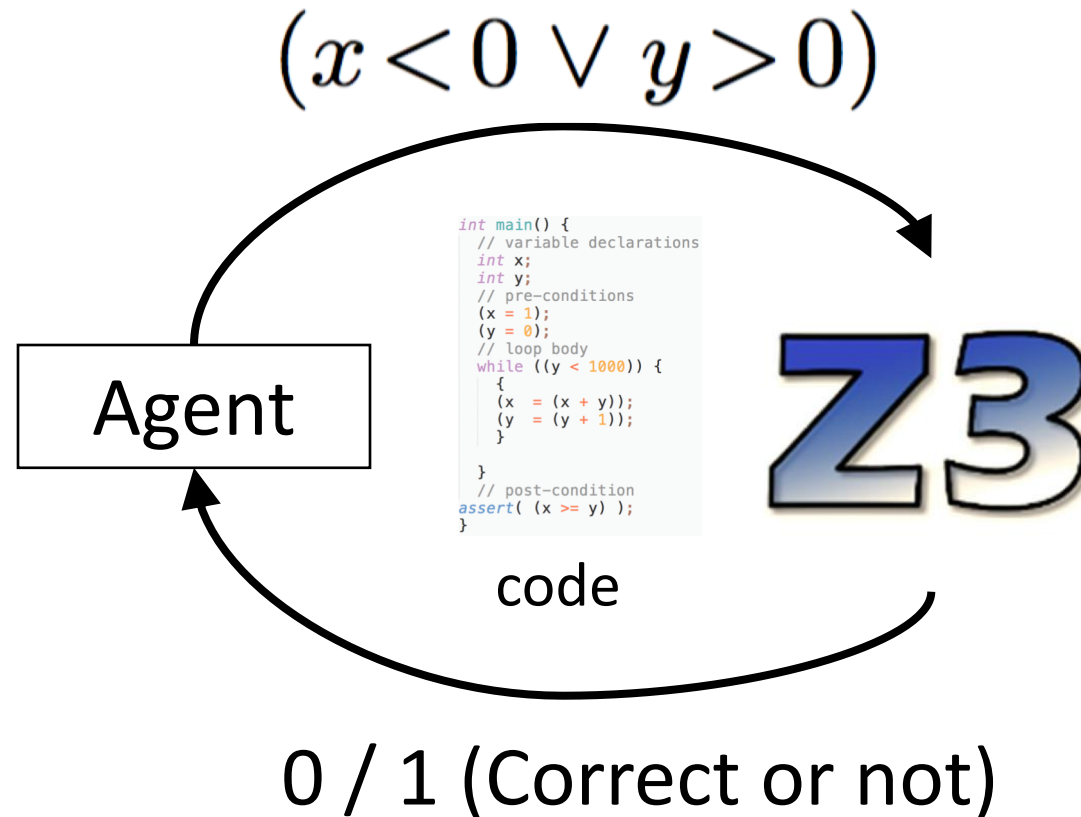
Difficulties of learning loop Invariant

1. Highly sparse and non-smooth reward



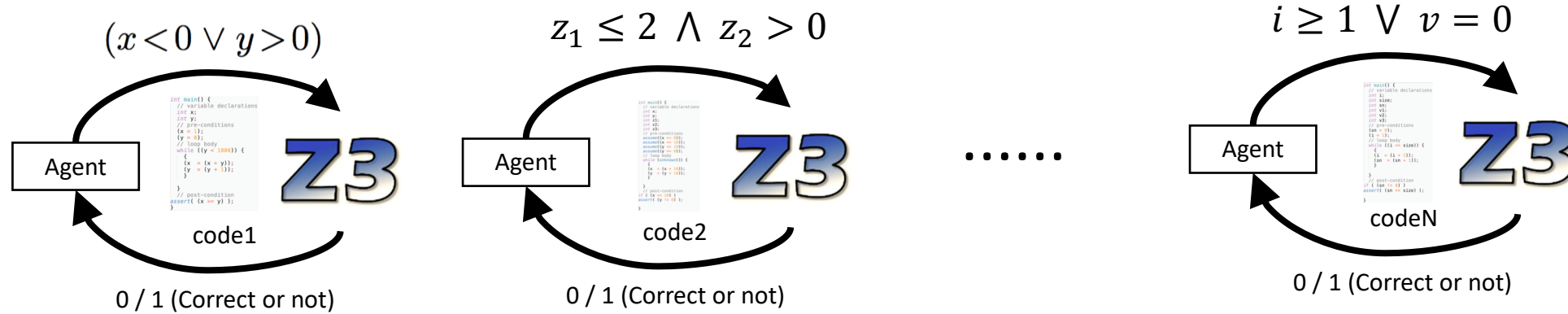
Difficulties of learning loop Invariant

1. Highly sparse and non-smooth reward



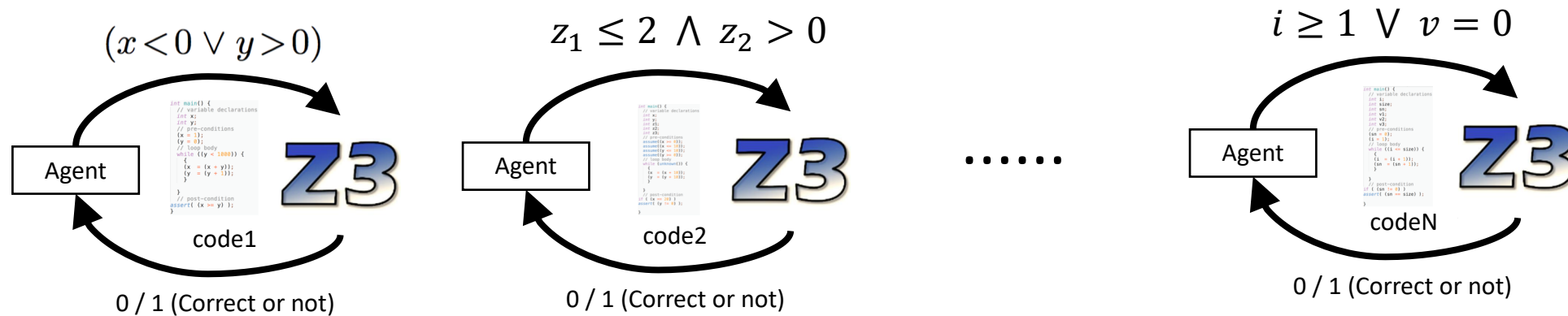
Difficulties of learning loop invariant

2. Generalization ability



Difficulties of learning loop invariant

2. Generalization ability



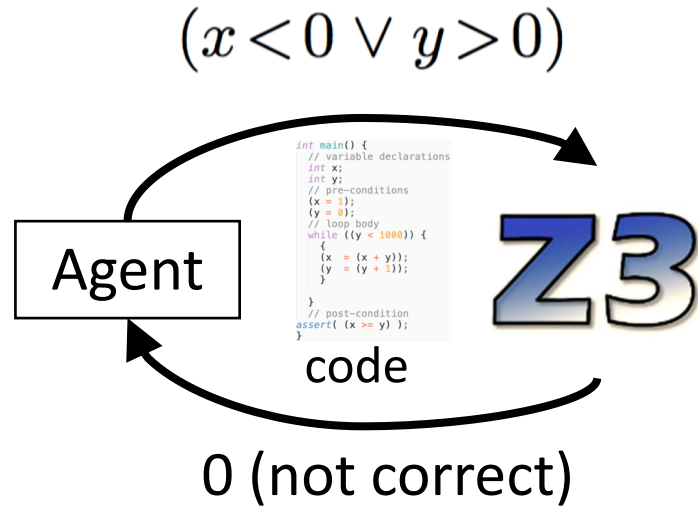
New code

```
int main() {  
  // variable declarations  
  int i;  
  int j;  
  int x;  
  int y;  
  // pre-conditions  
  {i = x};  
  {j = y};  
  // loop body  
  while ((x != 0)) {  
    {  
      x = (x - 1);  
      y = (y - 1);  
    }  
  }  
  // post-condition  
  if ( (i == j) )  
    assert( (y == 0) );  
}
```

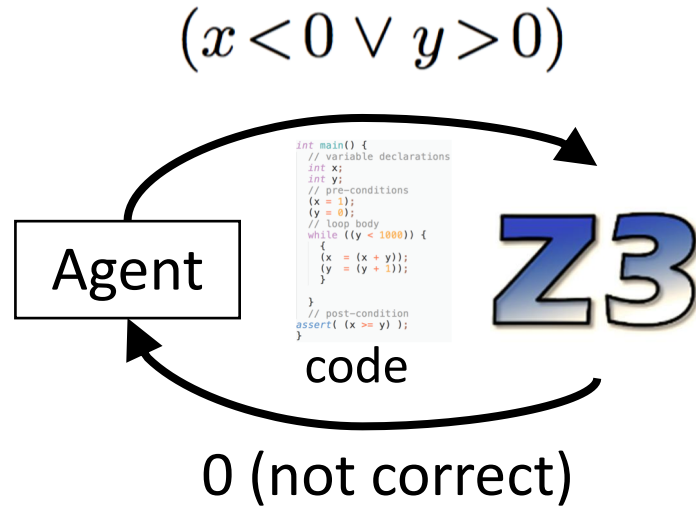
=>

Agent

Solution to sparsity and non-smoothness



Solution to sparsity and non-smoothness

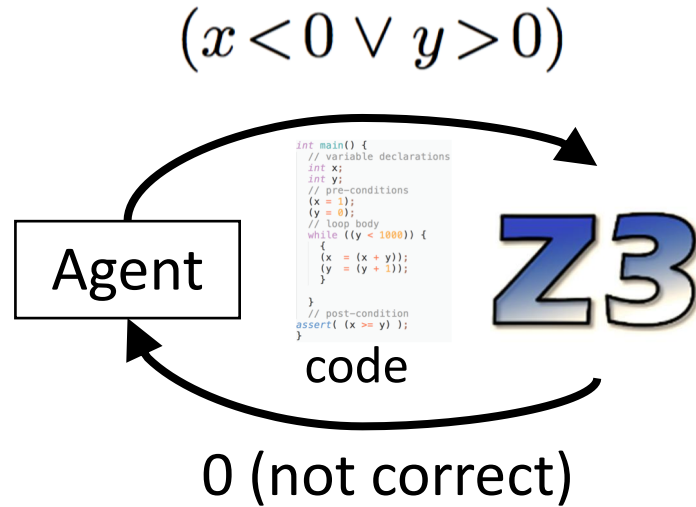


Counter-example: why am I wrong?

$$x = 1, y = -10$$

Solution to sparsity and non-smoothness

Collection of counter-examples:

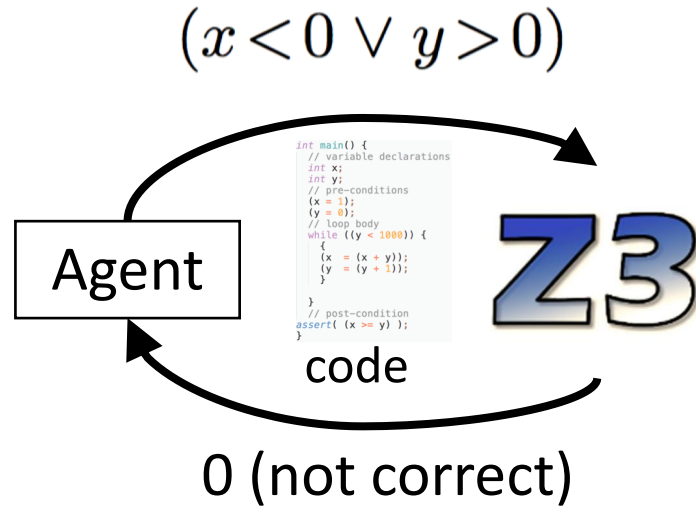


Counter-example: why am I wrong?

$$x = 1, y = -10$$

Solution to sparsity and non-smoothness

Collection of counter-examples:



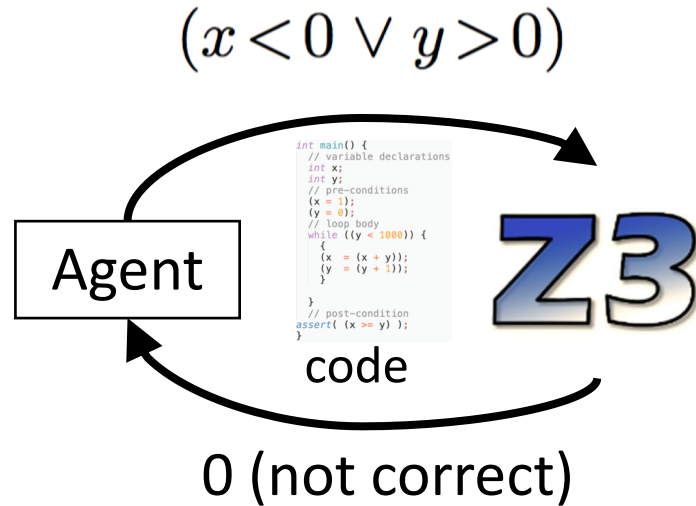
Counter-example: why am I wrong?

$$x = 1, y = -10$$

$x = 0, y = -2$	$x = 3, y = -2$	
$x = 0, y = -1$	$x = 3, y = -1$	
$x = 1, y = -1$	$x = 3, y = -1$	
	$x = 2, y = -2$	
	$x = 2, y = -1$	
	$x = 2, y = -1$	
		$x = 0, y = -4$
		$x = 0, y = -3$

Pre Inv Post

Solution to sparsity and non-smoothness



Counter-example: why am I wrong?

$$x = 1, y = -10$$

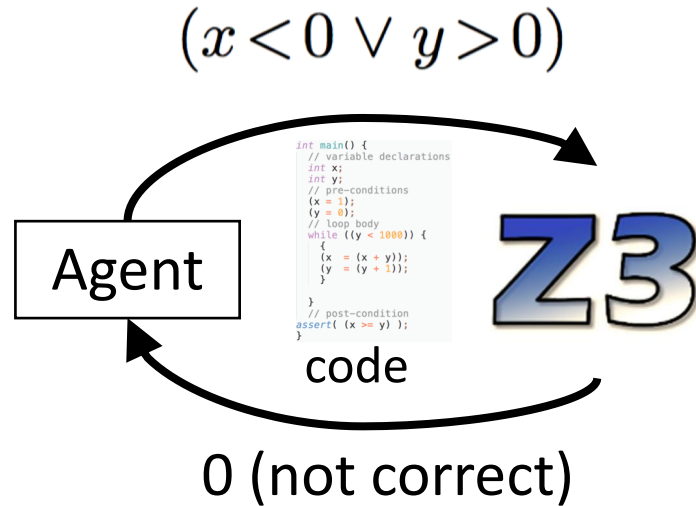
Collection of counter-examples:

$x = 0, y = -2$	$x = 3, y = -2$	
$x = 0, y = -1$	$x = 3, y = -1$	
$x = 1, y = -1$	$x = 3, y = -1$	
	$x = 2, y = -2$	
	$x = 2, y = -1$	
	$x = 2, y = -1$	
		$x = 0, y = -4$
		$x = 0, y = -3$

Pre Inv Post

- Smoothed reward

Solution to sparsity and non-smoothness



Collection of counter-examples:

$x = 0, y = -2$	$x = 3, y = -2$	
$x = 0, y = -1$	$x = 3, y = -1$	
$x = 1, y = -1$	$x = 3, y = -1$	
	$x = 2, y = -2$	
	$x = 2, y = -1$	
	$x = 2, y = -1$	
		$x = 0, y = -4$
		$x = 0, y = -3$

Pre Inv Post

Counter-example: why am I wrong?

$$x = 1, y = -10$$

- Smoothed reward
- Reduced Z3 calls

Solution to generalization

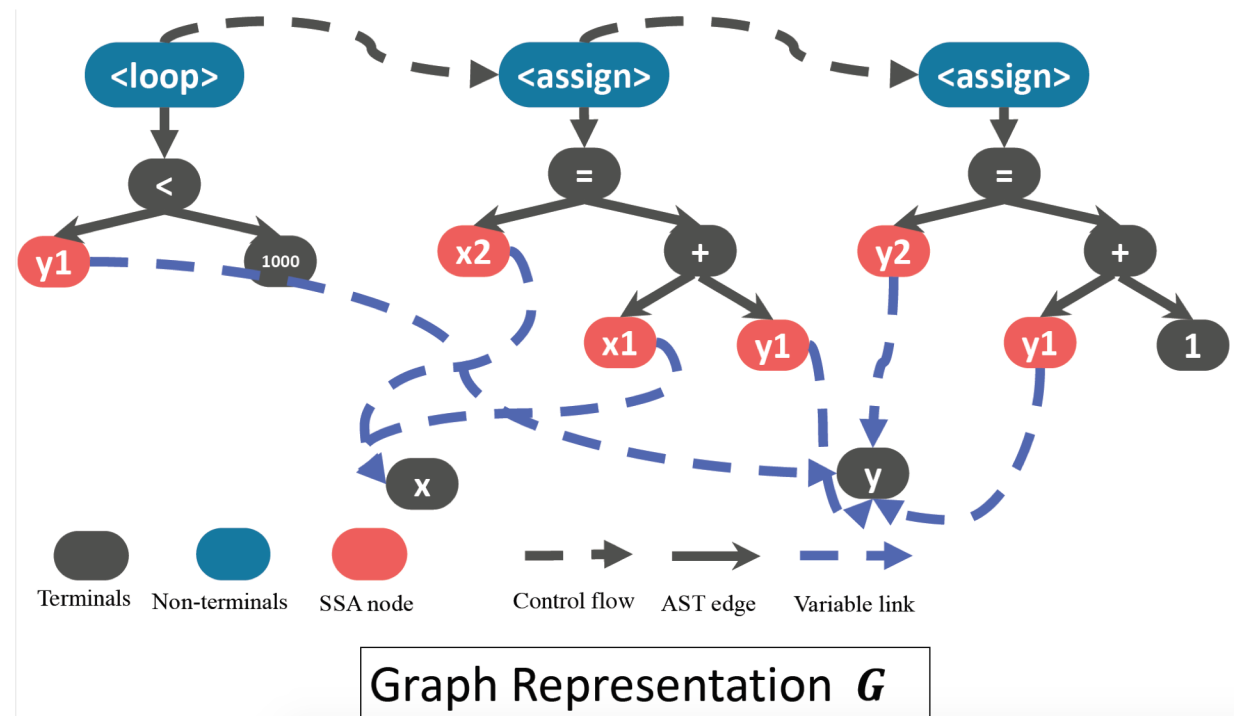
- Transferable graph representation of source code

```
while (y < 1000) {  
  x = x + y  
  y = y + 1  
}
```

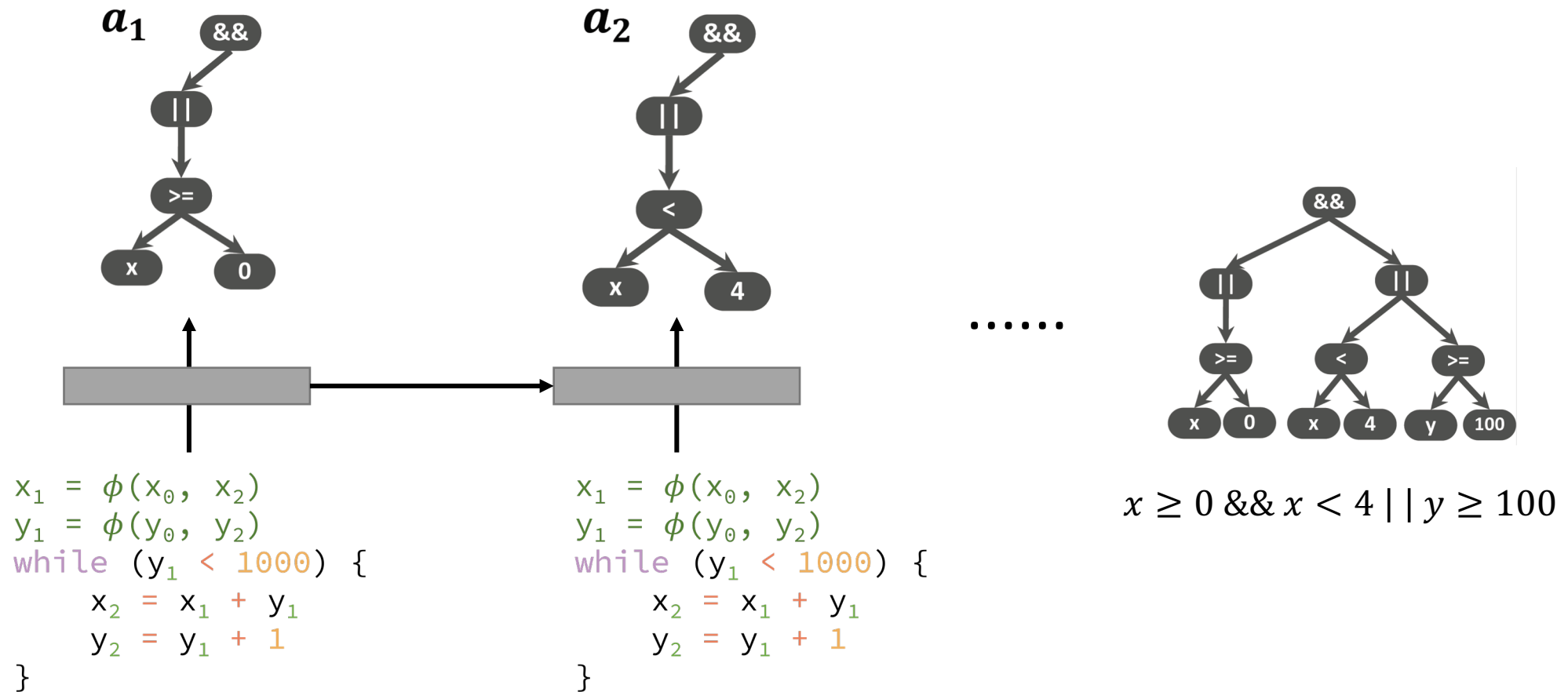
\Downarrow SSA Transformation

```
x1 =  $\phi$ (x0, x2)  
y1 =  $\phi$ (y0, y2)  
while (y1 < 1000) {  
  x2 = x1 + y1  
  y2 = y1 + 1  
}
```

\Rightarrow



Code2Inv: End-to-end learning framework



Experimental evaluation of Code2Inv

- We collect 133 benchmark programs



OOPSLA 2013, Dillig et al

POPL 2016, Garag et al

Experimental evaluation of Code2Inv

- We collect 133 benchmark programs



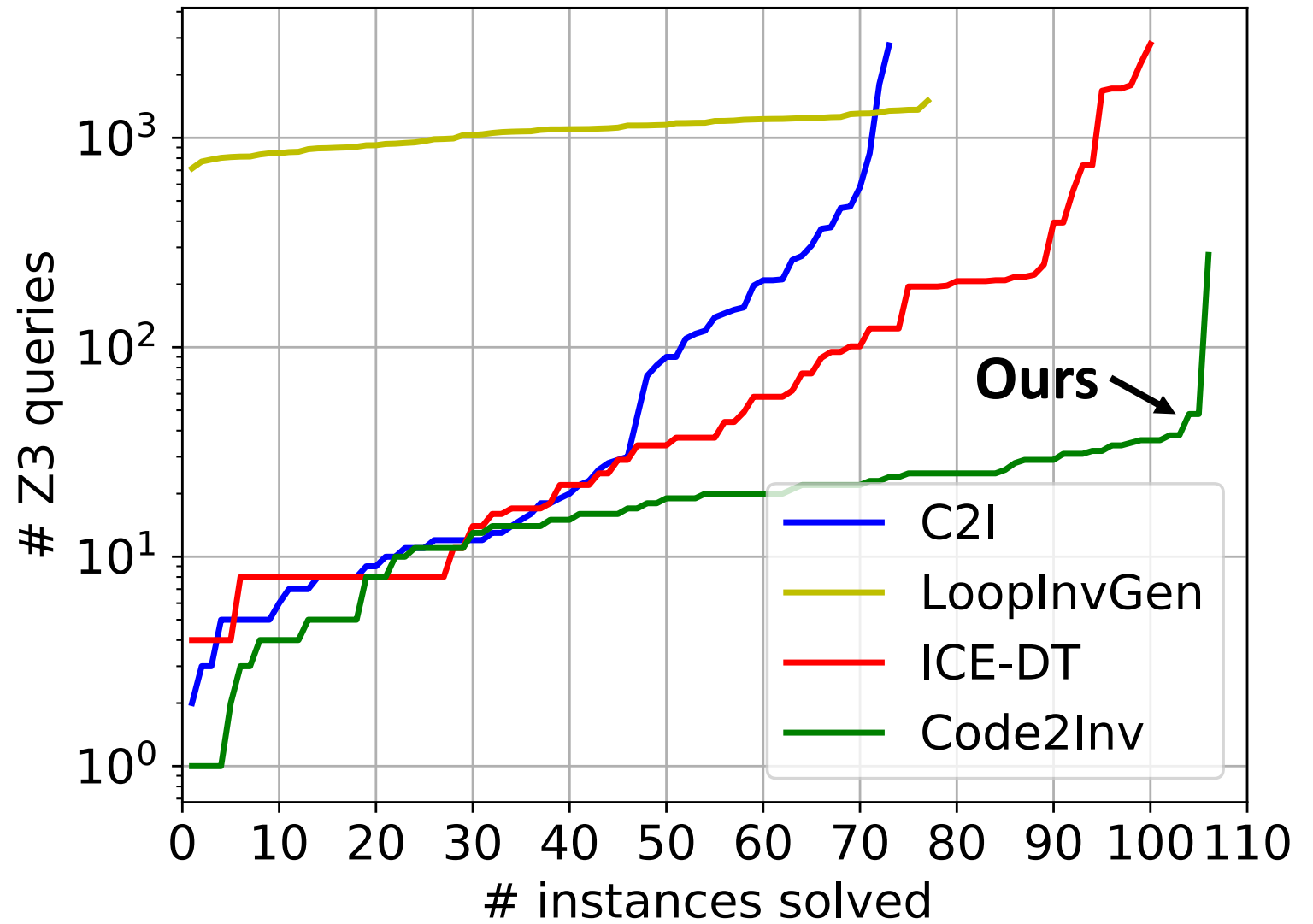
OOPSLA 2013, Dillig et al

POPL 2016, Garag et al

```
int main() {  
    int x = 0, y = 0;  
    while (*) {  
        if (*) {  
            x++;  
            y = 100;  
        } else if (*) {  
            if (x >= 4) {  
                x++;  
                y++;  
            }  
            if (x < 0) y--;  
        }  
    }  
    assert( x < 4 || y > 2);  
}
```

Code2Inv as an out-of-the-box solver

Solved more instances
with same # Z3 calls



Generalize to new programs

```
void main (int n) {  
    int x = 0  
  
    int m = 0  
  
    while (x < n) {  
        if (unknown()) {  
            m = x  
        }  
        x = x + 1  
    }  
    if (n > 0) {  
        assert (m < n)  
    }  
}
```

Generalize to new programs

```
void main (int n) {  
    int x = 0  
        ← int w = 0  
    int m = 0  
        ← int z = 0  
    while (x < n) {  
        if (unknown()) {  
            m = x  
        }  
        x = x + 1  
    }  
    if (n > 0) {  
        assert (m < n)  
    }  
}
```

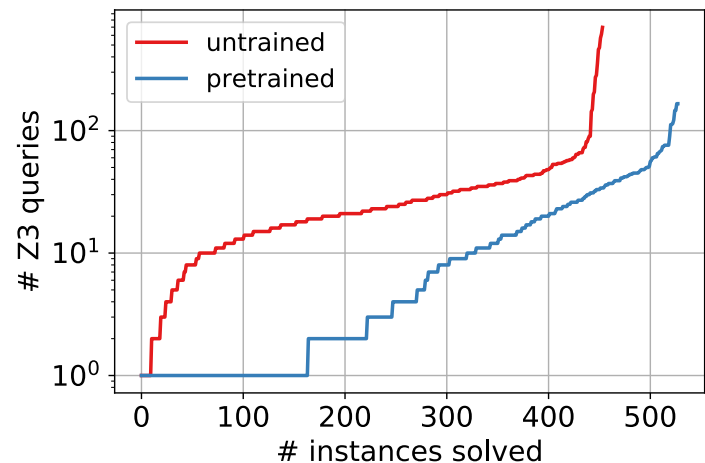
Generalize to new programs

```
void main (int n) {  
    int x = 0  
        ← int w = 0  
    int m = 0  
        ← int z = 0  
    while (x < n) {  
        ← z = z + 1  
        if (unknown()) {  
            m = x  
                ← z = m + 1  
        }  
        x = x + 1  
            ← w = m + x  
    }  
    if (n > 0) {  
        assert (m < n)  
    }  
}
```

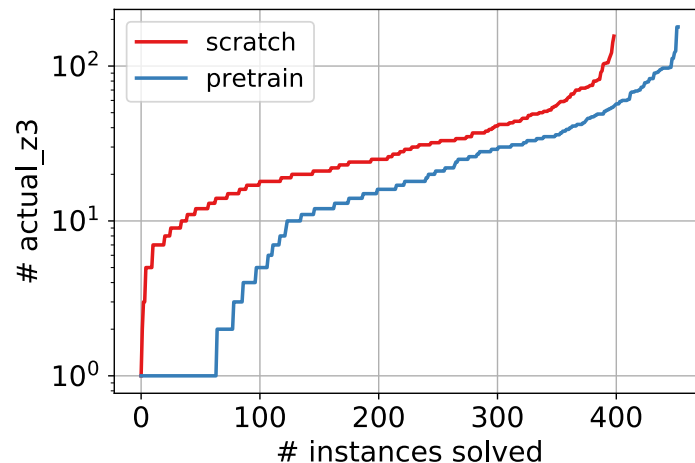
Generalize to new programs

```
void main (int n) {  
    int x = 0  
        ← int w = 0  
    int m = 0  
        ← int z = 0  
    while (x < n) {  
        ← z = z + 1  
        if (unknown()) {  
            m = x  
            ← z = m + 1  
        }  
        x = x + 1  
        ← w = m + x  
    }  
    if (n > 0) {  
        assert (m < n)  
    }  
}
```

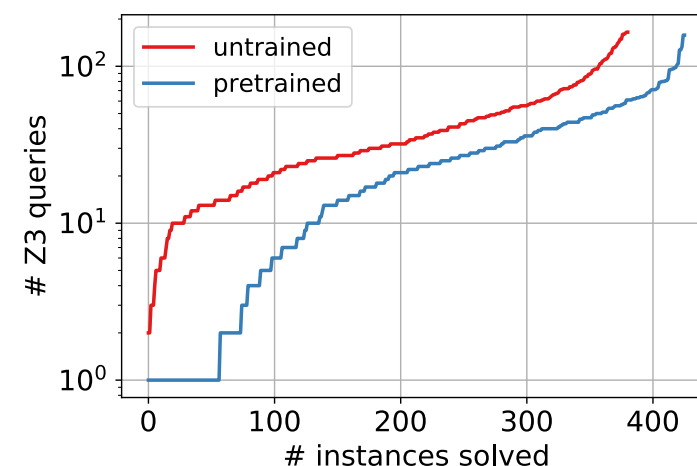
```
void main (int n) {  
    int x = 0  
    int w = 0  
    int m = 0  
    int z = 0  
    while (x < n) {  
        z = z + 1  
        if (unknown()) {  
            m = x  
            z = m + 1  
        }  
        x = x + 1  
        w = m + x  
    }  
    if (n > 0) {  
        assert (m < n)  
    }  
}
```

1 confounding variable



3 confounding variables



5 confounding variables

Generalization ability of Code2Inv

Poster session:

05:00 -- 07:00 PM

Room 210 & 230 AB #23